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On Some Prime Graphs

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Abstract:

A graph $G=(V,E)$ with n vertices is said to admit prime labeling if its vertices can be labeled with distinct positive integers not exceeding n such that the label of each pair of adjacent vertices are relatively prime. A graph G which admits prime labeling is called a prime graph. And a graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$. In this paper we investigate prime labeling for some graphs related to H- graph, ladder graph, comb graph and also we prove that comb graph is a strongly prime graph.

Keywords: Prime Labeling, Prime Graph, Strongly Prime Graph, H -Graph, Ladder Graph, Comb Graph .

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1. INTRODUCTION:

In this paper, we consider only simple, finite, undirected and non trivial graph $G=(V(G),E(G))$ with the vertex set V(G) and the edge set E(G). The set of vertices adjacent to a vertex u of G is denoted by N(u). For notations and terminology we refer to Bondy and Murthy [1].

Two integers a and b are said to be relatively prime if their greatest common divisor is 1. Relatively prime numbers play an important role in both analytic and algebraic number theory. The notion of a prime labeling was introduced by Roger Entringer and was discussed in a paper by Tout.A (1982 P 365-368) [7]. Many researchers have studied prime graph. For example Fu.H (1994 P 181-186) [3] have proved that path P_n on n vertices is a prime graph. Deresky.T (1991 P 359-369) [2] have proved that the C_n on n vertices is a prime graph. Lee.S (1998 P 59-67) [5] have proved that wheel W_n is a prime graph iff n is even. Around 1980 Roger Etringer conjectured that all trees having prime labeling which is not settled till today.

In [8] S.K.Vaidya and K.K.Kanani have proved the *Prime Labeling For Some Cycle Related Graph*. In [6] S.Meena and K.Vaithiligam have proved the *Prime Labeling For Some Helm Related Graph* (2013 P 1075-1085).

In [9] S.K.Vaidya and Udayan M.Prajapati have introduced Strongly prime graph and has proved the C_n, P_n and $K_{1,n}$ are Strongly prime graphs and W_n is a Strongly prime graph for every even integer $n \geq 4$, in *Some New Results On Prime Graph* (2012 P 99-104). In [10] R.Vasuki and A.Nagarajan have proved *Some Results On Super Mean Graphs* Vol.3 (2009), 82-96.For latest *Dynamic Survey On Graph Labeling* we refer to [4] (Gallian .J.A., 2009). Vast amount of literature is available on different types of graph labeling more than 1000 research papers have been published so far in last four decades.

Definition 1.1:

Let $G=(V(G),E(G))$ be a graph with p vertices. A bijection $f: V(G) \to \{1,2,\ldots\ldots p\}$ is called a prime labeling if for each edge $e=uv, \gcd\{f(u), f(v)\}=1$. A graph which admits prime labeling is called a prime graph.

Definition 1.2:

A graph G is said to be a strongly prime graph if for any vertex v of G there exists a prime labeling f satisfying $f(v)=1$.

Definition 1.3:

The H graph of a path P_n is the graph obtained from two copies of P_n with vertices $u_1,u_2,\ldots u_n$ and $v_1,v_2,\ldots v_n$ by joining the vertices $u_{\frac{n+1}{2}}$ and $v_{\frac{n+1}{2}}$ if n is odd and the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$ if n is even.

Definition 1.4:

The corona of a graph G on p vertices $v_1,v_2,\ldots v_p$ is the graph obtained from G by adding p new vertices $u_1,u_2,\ldots u_p$ and the new edges u_iv_i for $1 \leq i \leq p$ then it is denoted by $G \odot K_1$. And $G+K_1$ is a graph obtained from G by adding only one new vertex v_0 and join every vertices of G with v_0, then the new edges are v_0u_i, v_0v_i for $1 \leq i \leq n$.

Definition 1.5:

The product $P_2 \times P_n$ is called a ladder and it is denoted by L_n.

Definition 1.6:

The graph $P_n \odot K_1$ is called a comb C_{bn}. In this paper we investigate prime labeling for some graphs related to H- graph, ladder graph, comb graph and also we prove that comb graph is a strongly prime graph.

2. Prime Labeling of Some Graphs

Theorem 2.1:

The H-graph of path P_n is a prime graph.

Proof:

Let H_n be the H-graph with Vertex set is $\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n\}$.

The edge set is $E(H_n) = \{u_i u_{i+1}, v_i v_{i+1} / 1 \le i \le n-1\} \cup \{u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$ if n is odd} (or) $\{u_{\frac{n}{2}+1} v_{\frac{n}{2}}$ if n is even}.

Here $|V(H_n)| = 2n$

Define a labeling $f : V(H_n) \to \{1, 2, \ldots 2n\}$ by considering the following cases:

Case (i): When n is odd

$f(u_i) = i+1$ for $1 \le i \le n$,

$f(v_i) = n+i+1$ for $1 \le i < \frac{n+1}{2}$,

$f(v_i) = n+i$ for $\frac{n+1}{2} \le i \le n$,

$f\left(v_{\frac{n+1}{2}}\right) = 1$,

here $\gcd(f(u_i), f(u_{i+1})) = 1$ for $1 \le i \le n-1$,

$\gcd(f(v_i), f(v_{i+1})) = 1$ for $1 \le i < \frac{n-1}{2}$,

$\gcd(f(v_i), f(v_{i+1})) = 1$ for $\frac{n+1}{2} < i \le n$,

Since they are consecutive integers.

$$\gcd\left(f\left(v_{\frac{n-1}{2}}\right), f\left(v_{\frac{n+1}{2}}\right)\right) = \gcd\left(f\left(v_{\frac{n-1}{2}}\right), 1\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+3}{2}}\right)\right) = \gcd\left(1, f\left(v_{\frac{n+3}{2}}\right)\right) = 1,$$

$$\gcd\left(f\left(u_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+1}{2}}\right)\right) = \gcd\left(f\left(u_{\frac{n+1}{2}}\right), 1\right) = 1,$$

Clearly vertex label are distinct.

Thus labeling defined above gives a prime labeling for a graph H_n (*for n is odd*) .

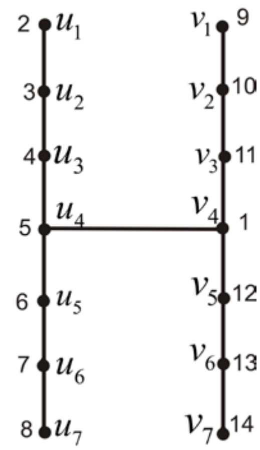

Figure 1: Prime labeling of H_7

Case (ii): When n is even

$f(u_i) = i+1$ for $1 \le i \le n$,

$f(v_i) = n+i+1$ for $1 \le i < \frac{n}{2}$,

$f(v_i) = n+i$ for $\frac{n}{2} < i \le n$,

$f(v_{\frac{n}{2}}) = 1$,

Now $\gcd(f(u_i), f(u_{i+1})) = 1$ for $1 \le i \le n-1$,

$\gcd(f(v_i), f(v_{i+1})) = 1$ for $1 \le i \le \frac{n}{2} - 1$,

Since they are consecutive integers.

$$\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(v_{\frac{n}{2}-1}\right)\right) = \gcd\left(1, f\left(v_{\frac{n}{2}-1}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(v_{\frac{n}{2}+1}\right)\right) = \gcd\left(1, f\left(v_{\frac{n}{2}+1}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(u_{\frac{n}{2}+1}\right)\right) = \gcd\left(1, f\left(u_{\frac{n}{2}+1}\right)\right) = 1,$$

$\gcd(f(v_i), f(v_{i+1})) = 1$ for $\frac{n}{2} < i \le n$,

Since it is a consecutive integer.

Clearly vertex label are distinct. Thus labeling defined above gives a prime labeling for a graph H_n (*for n is even*) .

Thus in both the cases f admits prime labeling. Hence H_n becomes a prime graph.

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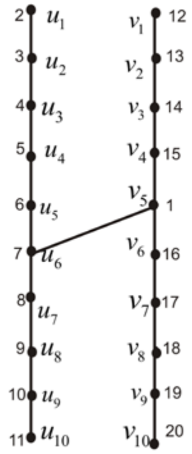

Figure 2: Prime labeling of H_{10}

Theorem 2.2:
The graph $G \odot K_1$ is a prime graph where G is a H-graph with n vertices.

Proof:
Let G be the graph with vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$. Let $u_1', u_2', \ldots u_n'$ and $v_1', v_2', \ldots v_n'$ be the corresponding new vertices, join $u_i u_i'$ and $v_i v_i'$ in G. we get the graph G_1 ie., $G \odot K_1$.

Now the vertex set of G_1 is $\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, u_1', u_2', \ldots u_n', v_1', v_2', \ldots v_n'\}$ and the edge set $E(G_1) = \{u_i u_{i+1}, v_i v_{i+1} / 1 \le i \le n-1\} \cup \{u_i u_i', v_i v_i' / 1 \le i \le n\}$

$$\cup \left\{ u_{\frac{n+1}{2}} v_{\frac{n+1}{2}} \ / \ if \ \ n \ is \ odd \right\} \quad \text{(or)}$$

$$\cup \left\{ u_{\frac{n}{2}+1} v_{\frac{n}{2}} \ / \ if \ \ n \ is \ even \right\}$$

Here $|V(G_1)| = 4n$

Define a labeling $f : V(G_1) \to \{1, 2, \ldots 4n\}$ by considering the following cases:

Case (i): When n is odd.

$f(u_i) = 2i+1$ for $1 \le i \le n$,

$f(u_i') = 2i$ for $1 \le i \le n$,

$f(v_i) = 2n+2i+1$ for $1 \le i < \frac{n+1}{2}$,

$f(v_i) = 2n+2i-1$ for $\frac{n+1}{2} < i \le n$,

$f\left(v_{\frac{n+1}{2}}\right) = 1$,

$f(v_i') = 2n+2i$ for $1 \le i \le n$,

here $\gcd(f(u_i), f(u_{i+1})) = \gcd(2i+1, 2i+3) = 1$ for $1 \le i \le n-1$,

$\gcd(f(v_i), f(v_{i+1})) = \gcd(2n+2i+1, 2n+2i+3) = 1$ for $1 \le i < \frac{n-1}{2}$,

$\gcd(f(v_i), f(v_{i+1})) = \gcd(2n+2i-1, 2n+2i+1) = 1$ for $\frac{n+1}{2} < i < n-1$,

Since they are all consecutive odd numbers.

$\gcd(f(u_i), f(u_i')) = \gcd(2i+1, 2i) = 1$ for $1 \le i \le n$,

$\gcd(f(v_i), f(v_i')) = \gcd(2n+2i+1, 2n+2i) = 1$ for $1 \le i \le n-1$,

Since they are consecutive integers.

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+1}{2}-1}\right) \right) = \gcd\left(1, f\left(v_{\frac{n+1}{2}-1}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+1}{2}+1}\right) \right) = \gcd\left(1, f\left(v_{\frac{n+1}{2}+1}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n+1}{2}}\right), f\left(u_{\frac{n+1}{2}}\right) \right) = \gcd\left(1, f\left(u_{\frac{n+1}{2}}\right)\right) = 1,$$

Clearly vertex labels are distinct.
Thus labeling defined above gives a prime labeling for a graph $G_1(\,for\, n\, is\, odd\,)$.

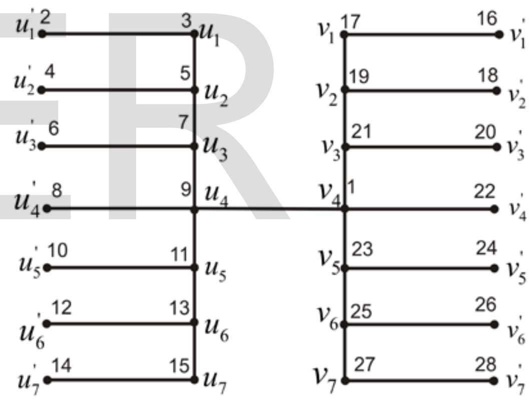

Figure 3: Prime labeling of $G \odot K_1$ $where\, G = H_7$

Case (ii): When n is even.

If n is even then we join the vertices $u_{\frac{n}{2}+1}$ and $v_{\frac{n}{2}}$.

Therefore in the above labeling f defined in case (i) we have to change only these labels $f\left(v_{\frac{n}{2}}\right) = 1$, $f(v_i) = 2n+2i+1$ for $1 \le i < \frac{n}{2}$, $f(v_i) = 2n+2i-1$ for $\frac{n}{2} < i \le n$.

Hence $\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(u_{\frac{n}{2}+1}\right) \right) = \gcd\left(1, f\left(u_{\frac{n}{2}+1}\right)\right) = 1,$

$$\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(v_{\frac{n}{2}+1}\right) \right) = \gcd\left(1, f\left(v_{\frac{n}{2}+1}\right)\right) = 1,$$

$$\gcd\left(f\left(v_{\frac{n}{2}}\right), f\left(v_{\frac{n}{2}-1}\right)\right) = \gcd\left(1, f\left(v_{\frac{n}{2}-1}\right)\right) = 1,$$

Now clearly vertex labels are distinct.
Thus labeling defined above gives a prime labeling for a graph $G_1(\text{for } n \text{ is odd})$. Thus $G \odot K_1$ is a prime graph.

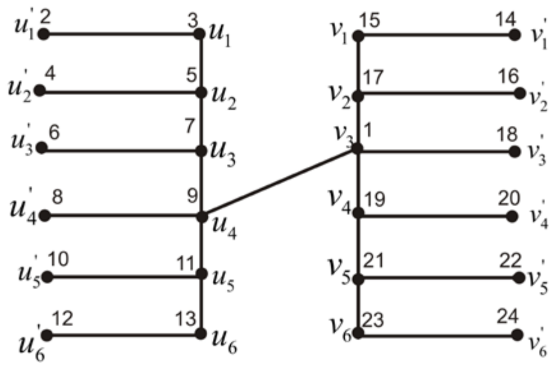

Figure 4: Prime labeling of $G \odot K_1 \, where \, G = H_6$

Theorem 2.3:

The graph obtained by identifying the central vertex of $K_{1,2}$ at each pendent vertex of a comb C_{bn} is a prime graph.

Proof:

Let P_n be the path $u_1, u_2, \ldots u_n$. Let v_i be a vertex adjacent to u_i, $1 \le i \le n$. The resultant graph is comb C_{bn}. Let x_i, w_i, y_i be the vertices of i'th copy of $K_{1,2}$ with w_i is the central vertex. Identify the vertex w_i with v_i, $1 \le i \le n$. We get the graph G whose vertex set is

$\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, x_1, x_2, \ldots x_n, y_1, y_2, \ldots y_n\}$.

The edge set

$= \{u_i u_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i, v_i\, x_i, v_i\, y_i \,/ 1 \le i \le n\}$

Here $|V(G)| = 4n$

Define a function $f : V(G) \to \{1, 2, \ldots 4n\}$ by

$f(u_i) = 4i - 3$ for $1 \le i \le n$,

$f(v_i) = 4i - 1$ for $1 \le i \le n$,

$f(x_i) = 4i - 2$ for $1 \le i \le n$,

$f(y_i) = 4i$ for $1 \le i \le n$,

Here

$\gcd(f(u_i), f(u_{i+1})) = \gcd(4i-3, 4i+1) = 1 \text{ for } 1 \le i \le n-1,$

as these two numbers are odd and their difference is 4

$\gcd(f(u_i), f(v_i)) = \gcd(4i-3, 4i-1) = 1,$

as these two numbers are consecutive odd numbers.

$\gcd(f(x_i), f(v_i)) = \gcd(4i-2, 4i-1) = 1$ and

$\gcd(f(y_i), f(v_i)) = \gcd(4i, 4i-1) = 1$

Since both are consecutive positive integers.

Clearly vertex label are distinct. Thus labeling define above gives a function f is a prime labeling of G. Thus G is a prime graph.

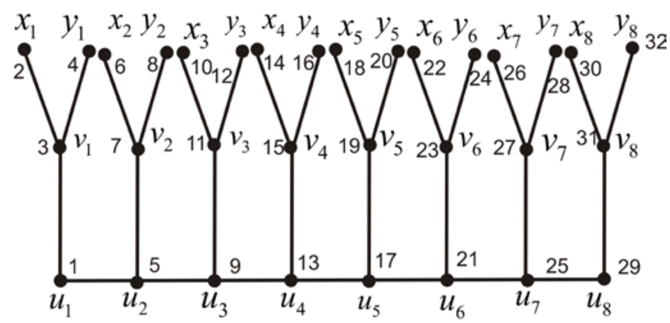

Figure 5: Prime labeling of Comb identifying the central vertex of $K_{1,2}$ **at each pendent vertex**

Theorem 2.4:

The graph $G \odot K_1$ is a prime graph where G is a Ladder graph with n vertices.

Proof:

Let $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$ be the two paths of equal length, join u_i and v_i, $1 \le i \le n$. The resultant graph is Ladder graph G. Add two new vertices x_i, y_i and join these vertices with u_i and v_i respectively, $1 \le i \le n$. we get the graph G_1 ie., $G \odot K_1$.

Now the vertex set is

$\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, x_1, x_2, \ldots x_n, y_1, y_2, \ldots y_n\}$.

The edge set is

$E(G_1) = \{u_i u_{i+1}, v_i\, v_{i+1} / 1 \le i \le n-1\} \cup \{u_i v_i, v_i\, y_i, u_i\, x_i \,/ 1 \le i \le n\}$

here $|V(G_1)| = 4n$

Define a function $f : V(G_1) \to \{1, 2, \ldots 4n\}$ by

$f(u_i) = 4i - 1$ for $1 \le i \le n$,

$f(v_i) = 4i - 3$ for $1 \le i \le n$,

$f(x_i) = 4i$ for $1 \le i \le n$,

$f(y_i) = 4i - 2$ for $1 \le i \le n$,

Here $\gcd(f(u_i), (u_{i+1})) = \gcd(4i-1, 4i+3) = 1$ for $1 \le i \le n-1$, as these two numbers are odd and their difference is 4.

$\gcd(f(v_i), f(v_{i+1})) = \gcd(4i-3, 4i+1) = 1,$

$\gcd(f(u_i), f(v_i)) = \gcd(4i-1, 4i-3) = 1,$

Since both are consecutive odd numbers.

$\gcd(f(u_i), f(x_i)) = \gcd(4i-1, 4i) = 1$ and

$\gcd(f(v_i), f(y_i)) = \gcd(4i-3, 4i-2) = 1,$

Since both are consecutive numbers.

Clearly vertex labels are distinct.

Thus a labeling defined above gives a prime labeling for a graph G_1. Thus $G \odot K_1$ is a prime graph.

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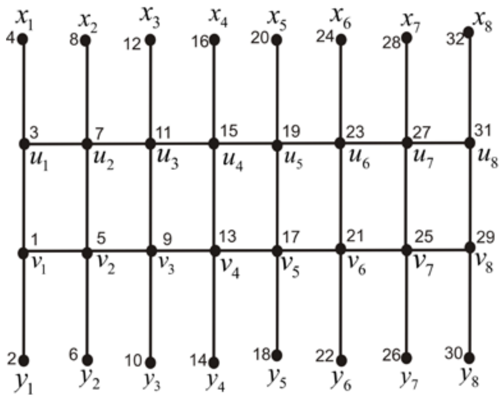

Figure 6: Prime labeling of $G \odot K_1$ *where* $G = L_8$

Theorem 2.5:

The graph $G+K_1$ is a prime graph where G is a H-graph with n vertices.

Proof:

Let G be the graph with vertices $u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n$. Let v_0 be the new vertex, join $v_0 u_i$ and $v_0 v_i$ where $1 \le i \le n$ in G. we get the graph G_1 ie., $G+K_1$.

Now the vertex set of G_1 is $\{u_1, u_2, \ldots u_n, v_1, v_2, \ldots v_n, v_0\}$.

The edge set

$$E(G_1) = \{u_i u_{i+1}, v_i v_{i+1} / 1 \le i \le n-1\} \cup \{v_0 u_i, v_0 v_i / 1 \le i \le n-1\}$$
$$\cup \left\{ u_{\frac{n+1}{2}} v_{\frac{n+1}{2}} \;/\; if\ \ n\ is\ odd \right\}$$
$$\cup \left\{ u_{\frac{n}{2}+1} v_{\frac{n}{2}} \;/\; if\ \ n\ is\ even \right\}$$

here $V|G_1| = 2n+1$.

Define a labeling $f : V(G_1) \to \{1, 2, 3, \ldots 2n+1\}$ by considering the following cases:

Case (i): When n is odd.

$f(v_0) = 1,$

$f(v_i) = \frac{n+1}{2} + i + 1$ for $1 \le i \le n,$

$f(u_i) = \frac{3n+1}{2} + i + 1$ for $1 \le i \le \frac{n+1}{2},$

$f(u_i) = i - \left(\frac{n-3}{2}\right)$ for $\frac{n+1}{2} \le i \le n,$

here $\gcd(f(v_0), f(v_i)) = \gcd(1, f(v_i)) = 1,$

$\gcd(f(v_0), f(u_i)) = \gcd(1, f(u_i)) = 1,$

$$\gcd(f(v_i), f(v_{i+1})) = \gcd\left(\frac{n+1}{2} + i + 1, \frac{n+1}{2} + i + 2\right) = 1$$

for $1 \le i \le n-1,$

$$\gcd(f(u_i), f(u_{i+1})) = \gcd\left(\frac{3n+1}{2} + i + 1, \frac{3n+1}{2} + i + 2\right) = 1$$

for $1 \le i \le \left(\frac{n+1}{2}\right) - 2,$

$$\gcd(f(u_i), f(u_{i+1})) = \gcd\left(i - \left(\frac{n-3}{2}\right), i - \left(\frac{n-3}{2}\right) + 1\right) = 1$$

for $\frac{n+1}{2} \le i \le n-1,$

Since these are all consecutive numbers.

and $\gcd\left(f\left(u_{\frac{n+1}{2}}\right), f\left(v_{\frac{n+1}{2}}\right)\right) = \gcd\left(2, f\left(v_{\frac{n+1}{2}}\right)\right) = 1,$

since $f\left(v_{\frac{n+1}{2}}\right)$ is odd.

Clearly vertex labels are distinct.

Thus the labeling defined above gives the prime labeling for a graph G_1(*for n is odd*).

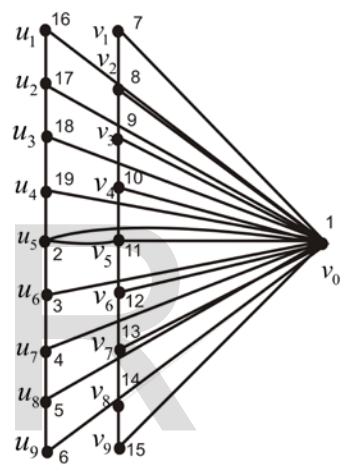

Figure 7: Prime labeling of $G+K_1$ *where* $G = H_9$

Case (ii): When n is even.

$f(v_0) = 1,$

$f(u_i) = \frac{3n}{2} + i + 1$ for $1 \le i < \frac{n}{2} + 1,$

$f(u_i) = i - \left(\frac{n-2}{2}\right)$ for $\frac{n}{2} + 1 \le i \le n,$

$f(v_i) = \frac{n}{2} + i + 1$ for $1 \le i \le n,$

Similar to case(i) here also $\gcd(f(u_i), f(u_{i+1})) = 1,$

$\gcd(f(v_i), f(v_{i+1})) = 1,$

$\gcd(f(v_0), f(u_i)) = 1,$ $\gcd(f(v_0), f(v_i)) = 1$ and

$\gcd\left(f\left(u_{\frac{n}{2}+1}\right), f\left(v_{\frac{n}{2}}\right)\right) = 1.$

Thus $G+K_1$ is a prime graph.

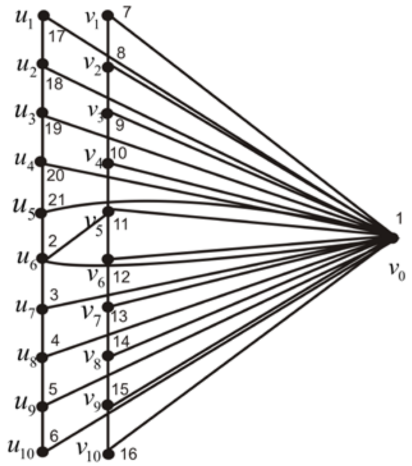

Figure 8: Prime labeling of $G+K_1 where G = H_{10}$

3. STRONGLY PRIME GRAPHS

Theorem 3.1:

The Comb graph C_{bn} is a strongly prime graph.

Proof:

Let C_{bn} be the Comb graph with vertex set $\{v_1, v_2, \ldots v_n, v_1^{'}, v_2^{'}, \ldots v_n^{'}\}$.

Let $E(C_{bn})$ be the edge set of the comb graph is $E(C_{bn}) = \{v_i v_i^{'} / 1 \le i \le n\} \cup \{v_i v_{i+1} / 1 \le i \le n-1\}$.

Here $V|C_{bn}| = 2n$, where n is a positive integer.

If v is any arbitrary vertex of C_{bn} then we have the following possibilities.

Case (i): When v is of degree 2,3.

If $v = v_j$ for some $j \in \{1,2,3,\ldots n\}$ then the function $f : V(C_{bn}) \to \{1,2,3,\ldots 2n\}$ defined by

$$f(v_i) = \begin{cases} 2n+2i-2j+1 & if \;\; i = 1,2,\ldots j-1; \\ 2i-2j+1 & if \;\; i = j+1, j+2, \ldots n, \end{cases}$$

$f(v_j) = 1$,

and $$f(v_i^{'}) = \begin{cases} 2n+2i-2j+2 & if \;\; i = 1,2,\ldots j-1; \\ 2i-2j+2 & if \;\; i = j+1, j+2, \ldots n, \end{cases}$$

$f(v_j^{'}) = 2$.

$$\gcd(f(v_i), f(v_{i+1})) = \gcd(2n+2i-2j+1, 2n+2i-2j+3) = 1 \quad \text{for } 1 \le i \le j-2,$$

$$\gcd(f(v_i), f(v_{i+1})) = \gcd(2i-2j+1, 2i-2j+3) = 1 \quad \text{for } 1 \le i \le n-1,$$

Since they are all consecutive odd numbers.

$$\gcd(f(v_i), f(v_i^{'})) = \gcd(2n+2i-2j+1, 2n+2i-2j+2) = 1 \quad \text{for } 1 \le i \le j-1,$$

$$\gcd(f(v_i), f(v_i^{'})) = \gcd(2i-2j+1, 2i-2j+2) = 1 \quad \text{for } 1 \le i \le n.$$

Since they are consecutive integers.

Clearly vertex label are distinct.

Thus C_{bn} is a prime labeling with $f(v) = f(v_j) = 1$. Thus f admits prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of degree 2,3 in C_{bn}.

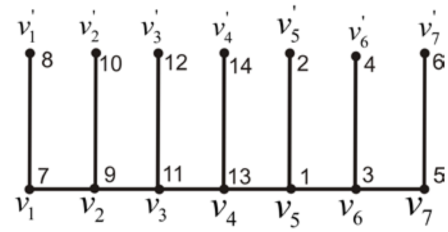

Figure 9: Strongly prime labeling of Comb C_{b7} **(** $v = v_5$ **)**

Case (ii): When v is of degree 1.

Let $v = v_j^{'}$ for some $j \in \{1,2,3,\ldots n\}$. let f_2 be the labeling obtained from f in case (i) by interchanging the labels $f(v_j)$ and $f(v_j^{'})$ and for all other remaining vertices $f_2(v) = f(v)$.Then the resulting labeling f_2 is a prime labeling as well as it is possible to assign label 1 to any arbitrary vertex of C_{bn}. Thus from all the cases described above C_{bn} is a strongly prime graph.

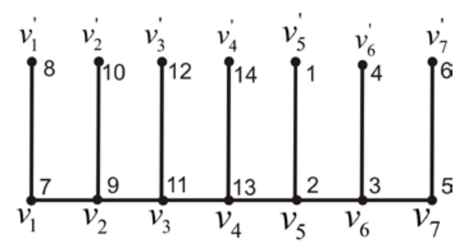

Figure 10: Strongly prime labeling of Comb C_{b7} **(** $v = v_5^{'}$ **)**

4. CONCLUDING REMARKS

The prime numbers and their behavior are of great importance as prime numbers are scattered and there are arbitrarily large gaps in the sequence of prime numbers. If these characteristics are studied in the frame work of graph theory then it is more challenging and exciting as well. Here we investigate several results on prime graphs and we prove that comb graph is a strongly prime graph.

REFERENCES

[1] J.A.Bondy and U.S.R.Murthy, "Graph Theory and Applications" (North-Holland), Newyork, 1976.
[2] To Dretskyetal "on Vertex Prime labeling of graphs in graph theory", Combinatories and applications vol.1 J.Alari (Wiley. N.Y.) pp 299-359, 1991.
[3] H.C. Fu and K.C.Huany "on Prime labeling Discrete Math", 127 pp 181-186, 1994
[4] Gallian J. A, "A dynamic survey of graph labeling", The Electronic Journal of Combinations 16 # DS6, 2009.
[5] S.M.lee, L.Wui and J.Yen "on the amalgamation of Prime graphs Bull", Malaysian Math.Soc.(Second Series) 11, pp 59- 67, 1988.
[6] S. Meena and K. Vaithilingam, "Prime labeling for some helm related graphs", International Journal of Innovative Research in Science, Engineering and Technology vol.2,Issue 4,April 2013.
[7] A.Tout A.N.Dabboucy and K.Howalla "Prime labeling of graphs". Nat. Acad. Sci letters 11 pp 365-368, 1982.
[8] S.K.Vaidya and K.K.Kanmani "Prime labeling for some cycle related graphs", Journal of Mathematics Research, vol.2. No.2.pp 98-104, May 2010.
[9] S.K.Vaidya and Udayan M.Prajapati "Some new results on prime graph", Open Journal of Discrete Mathematics (2012 P 99-104).
[10] R. Vasuki and A. Nagarajan" Some Results on Super Mean Graphs", International J.Math. Combin. Vol.3 (2009), 82-96.